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Cambridge Pre-U Certificate

MATHEMATICS **9794/03**

Paper 3 Applications of Mathematics **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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1				With no working shown allow only correct answers.
		$S_{xy} = 68.8 - \dfrac{24.7 \times 33.9}{10} = -14.933$	M1	Use of formula for numerator.
		$S_{xx} = 74.93 - \dfrac{24.7^2}{10} = 13.921$	M1	Use of formula for either term in denominator.
		$S_{yy} = 149.71 - \dfrac{33.9^2}{10} = 34.789$		
		$r = \dfrac{-14.933}{\sqrt{13.921 \times 34.789}} = -0.678(56...) \approx -0.679$	M1 A1 [4]	Use of formula for r. c.a.o. Must see unrounded value to at least 4sf first.
2	**(i)**	$W \sim \text{N}(1.349, 0.236^2)$	M1	Standardising.
		$\text{P}(W < 1.100) = \text{P}\left(Z < \dfrac{1.100 - 1.349}{0.236}\right)$		
		$= \Phi(-1.055)$	A1	C.a.o.
		$= 1 - 0.8543$	M1 B1	1 – … to deal with negative z value. Correct table look-up, including use of difference columns: e.g. 0.8543 seen, ft abs(c's z-value).
		$= 0.1457$	A1 [5]	ft c's negative z-value. Note: $z = -1.05 \Rightarrow p = 1 - 0.8531 = 0.1469$ $z = -1.06 \Rightarrow p = 1 - 0.8554 = 0.1446$
	(ii)	$\Phi^{-1}(0.9) = 1.282$	B1	Correct table look-up.
		$w = 1.349 + 1.282 \times 0.236$ $= 1.6515\ldots \approx 1.65$ (kg)	M1 A1 [3]	C.a.o.
3	**(i)**			With no working shown allow only correct answers.
		$\sum x = 20 \times 0.7 + 4 = 18$	M1 A1	Find Σx for $n = 20$. C.a.o.
		$\sum x^2 = 20(0.9^2 + 0.7^2) + 1 + 4 + 1 = 32$	M1 A1	Find Σx^2 for $n = 20$. C.a.o. Allow M1A0 for either 25.19 or 31.19 seen, i.e. for use of unbiased sd provided used consistently later.
		$\bar{x} = \dfrac{18}{25} = 0.72$ kg	B1	C.a.o.
		$s = \sqrt{\dfrac{32}{25} - 0.72^2}$	M1	Use of correct formula for standard deviation; may be implied.
		$\therefore s = \sqrt{0.7616} = 0.872(69...) \approx 0.873$ kg	A1 [7]	C.a.o. Allow M1A0 for unbiased sd (0.8715...) provided used consistently earlier.

	(ii)	$y = 65 + 25x$ $\bar{y} = 65 + 25 \times 0.72 = 83$ $s_y = 25 \times 0.873 = 21.825$	 M1 A1 M1 A1 [4]	y = total payment, x = goals scored. ft c's mean in (i). ft c's sd in (i). Accept 21.80 or 21.82 – 21.83, i.e. to nearest penny or better.
4	**(i)** **(a)**	$X \sim$ B(10, 0.2) P($X \leqslant 3$) = 0.8791	 B1 [1]	
	(b)	P($X = 3$) = 0.8791 – 0.6778 = 0.2013	M1 A1 [2]	Use of tables for P($X \leqslant 3$) – P($X \leqslant 2$) or formula for P($X = 3$). C.a.o.
	(c)	P($X \geqslant 1$) = 1 – 0.1074 = 0.8926	M1 A1 [2]	Attempt 1 – P($X \leqslant 0$) using tables or formula for P($X = 0$). C.a.o
	(ii)	$X \sim$ B(n, 0.2) Require P($X \geqslant 1$) $\geqslant 0.95$ i.e. require n s.t. P($X \geqslant 1$) = $1 - 0.8^n \geqslant 0.95$ or s.t. $0.8^n \leqslant 0.05$	 M1	
		ALT version1, by trial and error: If $n = 13$, P($X \geqslant 1$) = 1 – 0.0549… = 0.9450… < 0.95	 A1	Either trial: $n = 13$ or 14. The outcome of the trial must be shown explicitly.
		If $n = 14$, P($X \geqslant 1$) = 1 – 0.0439… = 0.9570… > 0.95 $\therefore n = 14$	A1 [3]	Both trials shown explicitly and the conclusion stated.
		ALT version 2, by logs: $n \log 0.8 \leqslant \log 0.05$ $\therefore n \geqslant \dfrac{\log 0.05}{\log 0.8} = \dfrac{-1.3010}{-0.0969}$ or $\dfrac{-2.9957}{-0.2231} = 13.425$	 A1 A1	Takes logs, any base, and make n the subject. Correct value and conclusion.

5	$\frac{8!}{2!2!}(=10080)$	B1	Arrangements of SEPARATE.
	ALT version 1:		
	$\frac{6!}{2!}\times {}^7C_2\,(=360\times 21=7560)$	B1	Arrangements of SEPRTE and place As apart.
	$p=\frac{6!/(2!)\times {}^7C_2}{8!/(2!2!)}$	M1	Ratio of attempts at relevant expressions.
	$=\frac{3}{4}$	A1 [4]	C.a.o.
	ALT version 2:		
	$\frac{7!}{2!}(=2520)$	B1	Arrangements with As together.
	$p=1-\frac{7!/(2!)}{8!/(2!2!)}$	M1	1 – ratio of attempts at relevant expressions.
	$=\frac{3}{4}$	A1	C.a.o.
6	$\mathrm{P}(B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}=\frac{1/4}{3/4}=\frac{1}{3}$	B1	Using independence of A and B.
	Either		Allow the use of a Venn diagram or a tree diagram.
	$\mathrm{P}(A'\cap B)=\mathrm{P}(A')\times\mathrm{P}(B)=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$	M1	Product using independence or partition of B.
	Or		
	$\mathrm{P}(A'\cap B)=\mathrm{P}(B)-\mathrm{P}(A\cap B)=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$	A1	C.a.o.
	Either		
	$\mathrm{P}(A'\cap B')=\mathrm{P}(A')\times\mathrm{P}(B')=\frac{1}{4}\times\frac{2}{3}=\frac{1}{6}$	M1	Product of complements or complement of union or partition of A'.
	Or		
	$\mathrm{P}(A'\cap B')=1-\mathrm{P}(A\cup B)=1-\left(\frac{3}{4}+\frac{1}{3}-\frac{1}{4}\right)=\frac{1}{6}$	A1	C.a.o.
	Or		
	$\mathrm{P}(A'\cap B')=\mathrm{P}(A')-\mathrm{P}(A'\cap B)=\frac{1}{4}-\frac{1}{12}=\frac{1}{6}$	[5]	
7	$15a=-\mathrm{F}$	M1	N2 applied.
	$F=\mu N=0.017\times 15\times 10\ (=2.55)$	B1	Limiting friction and $N=mg$.
	$\therefore\ a=-0.017\times 10=-0.17\ (\mathrm{ms}^{-2})$	A1	C.a.o.
	$0^2=4^2-2\times 0.17\times s$	M1	Use of appropriate '*suvat*' equation.
	$\therefore s=\frac{16}{2\times 0.17}=47.058...\approx 47.1\ (\mathrm{m})$	A1 [5]	Ft c's a.
8 (i)	At max height $v_y=0$ i.e. $U\sin\theta-10t=0$	M1	Use of condition for max height.

		$\therefore U \times \frac{12}{13} - 10 \times 2.4 = 0$		Allow M1A0 for sin / cos error here only; but absence is M0.
		$\therefore U = 26$ (ms^{-1})	A1	C.a.o.
		Max $y = Ut\sin\theta - \frac{1}{2}gt^2$		
		$y = 26 \times 2.4 \times \frac{12}{13} - 5 \times 2.4^2$	M1	Use of formula for y.
		$\therefore y = 28.8$ (m)	A1	Ft c's U.
			[4]	
	(ii)	$\cos\theta = \frac{5}{13}$	B1	
		ALT version 1: At horizontal range $t = 4.8$ (sec)	B1	
		$R = Ut\cos\theta$		
		$\therefore R = 26 \times 4.8 \times \frac{5}{13}$	M1	Use of formula for x
		$\therefore R = 48$ (m)	A1	Ft c's U.
			[4]	
		ALT version 2:		
		$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{12}{13} \times \frac{5}{13}$	B1	
		$\therefore R = \frac{U^2}{g}\sin 2\theta$		
		$\therefore R = \frac{26^2}{10} \times \frac{2 \times 12 \times 5}{13^2}$	M1	Use of formula for R.
		$\therefore R = 48$ (m)	A1	Ft c's U.
9	**(i)**	$0^2 = 165^2 + 2 \times a \times 1237.5$	M1	Use of appropriate '*suvat*' equation.
		$\therefore a = -11$ (m s^{-2})	A1	Correct outcome. Must be either negative or suitably qualified.
		$0.01 \times (-11) = -0.01 \times 10 - R$	M1	Use of N2; signs correct.
		$\therefore R = 0.01$ (N)	A1	Ft c's a provided it is negative.
			[4]	
	(ii)	$0.01 \times a = 0.01 \times 10 - 0.01$	M1	Use of N2; signs adjusted correctly.
		$\therefore a = 9$ (ms^{-2})	A1	Ft c's R.
		$1237.5 = 0 + \frac{9}{2}t^2$	M1	Depends on the previous M mark. Use of appropriate '*suvat*' equation.
		$\therefore t = \sqrt{275} = 16.583... \approx 16.6$ (s)	A1	Correct outcome. C.a.o.
			[4]	

10	**(a)**	C of M: $\lambda m \times 2u - m \times u = \lambda m \times \frac{11u}{10} + m \times \frac{7u}{2}$	M1	In direction of B. 4 terms; condone sign errors.
		$\therefore \frac{9\lambda}{10} = \frac{9}{2} \therefore \lambda = 5$	A1	C.a.o.
		NEL: $e(2u + u) = \frac{7u}{2} - \frac{11u}{10}$	M1	Condone sign errors.
		$\therefore 3e = \frac{24}{10} \therefore e = \frac{8}{10}$	A1 [4]	C.a.o.
	(b)	$3\mathbf{v} - 3(2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) = 6\mathbf{i} - 6\mathbf{j} - 9\mathbf{k}$ $\therefore \mathbf{v} = (2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + (6\mathbf{i} - 6\mathbf{j} - 9\mathbf{k})/3$ $= 4\mathbf{i} + \mathbf{j} - 5\mathbf{k}$	M1 A1 A1 [3]	Use of 'Impulse = change in mom.' All signs/terms correct. **v** fully correct. C.a.o.
11	**(i)** **(a)**	At A: weight, normal contact, tension and friction shown correctly. At B: weight and tension shown correctly.	B1 B1 [2]	Friction must be up the slope. Tension should be the same as at A.
	(b)	Values for g, $\sin\theta$ and $\cos\theta$, and $m_2 = \frac{1}{2}m_1$ can be substituted at any stage.		
		At A: $F + T = m_1 g \sin\theta = 6m_1$	B1	Resolve up the slope.
		At A: $N = m_1 g \cos\theta = 8m_1$	B1	Resolve perpendicular to the slope. Can be awarded in Part **(iii)** if not here.
		At A: $F = \mu N = \mu \times 8m_1$	B1	Limiting friction. Can be awarded in Part **(iii)** if not here.
		At B: $T = m_2 g = 5m_1$	B1	Resolve vertically.
		$\mu \times 8m_1 + 5m_1 = 6m_1$	M1	Eliminate F, N and T.
		$\therefore \mu = \frac{1}{8}$	A1 [6]	C.a.o. SR: Allow max 5/6 when T is eliminated as follows: $T + F - m_1 g\sin\theta = T - m_2 g$ or equivalent, i.e setting '0 = 0'.
	(ii)	Values for g, $\sin\theta$ and $\cos\theta$, and $m_2 = m_1$ can be substituted at any stage.		
		At A: $m_1 a = T - F - m_1 g \sin\theta = T - F - 6m_1$	B1	N2 applied up the slope. Friction must now be down the slope.
		At B: $m_2 a = m_2 g - T$ or $m_1 a = 10m_1 - T$	B1	N2 applied downwards.
		$2m_1 a = 10m_1 - \frac{1}{8} \times 8m_1 - 6m_1 = 3m_1$	M1	Eliminate T, N and F using expressions from above.
		$\therefore a = 1.5$ (ms^{-2})	A1 [4]	Ft c's μ, provided $a \geqslant 0$ ($0 \leqslant \mu \leqslant \frac{1}{2}$).